Abstract: This paper presents a method of economic factorial analysis based on the Divisia index extended to interconnected factors. We verify the applicability of the presented method to financial market research by examining fluctuations of the Warsaw Stock Exchange WIG Index (WIG). We consider four main factors of WIG changes: the GDP growth, the PLN/EUR rate, the S&P500 and the unemployment rate. Due to computational reasons we apply the transformation that produces variables in the bigger the better form. We use quarterly data from the time interval between 2003 and 2014 divided into periods of bull and bear market. All considered variables are assumed to change linearly between quarters. The main conclusion is that during market prosperity, GDP and S&P500 changes exhibit the strongest influence on WIG changes.

Keywords: Factorial analysis, Divisia index, interconnected factors, Warsaw Stock Exchange Index analysis

JEL: C43, G10
1. Introduction

The main aim of the paper is to verify the utility of the extended Divisia price index in financial data analysis. On one hand Divisia’s methodology is connected with continuous time price models and its application is naturally limited. On the other hand the analysed economic processes can be approximated by continuous functions between observation moments (linear, exponential and others) and thus, Divisia’s method seems to be applicable. As the title of the paper suggests, our work is an application of Divisia’s method on the Warsaw Stock Exchange and it can be treated as its illustration. Nevertheless, the added value of the paper is the mathematical formula for WIG changes derived from Divisia’s general formula for eight factors. In particular, we intend to examine fluctuations of the Warsaw Stock Exchange WIG Index (WIG) by considering four main factors of WIG changes: the GDP growth, the PLN/EUR rate, the S&P500 and the unemployment rate. Having linear approximations of these processes for quarterly data, we are going to describe WIG changes using the extended Divisia index with interconnected factors, i.e. assuming that the above-mentioned factors are related. Obviously, the problem of WIG fluctuations measurement is already known and it is considered in many studies by many authors. Nevertheless, the presented Divisia approach is quite unique: firstly, it is a continuous time approach, secondly, it takes into account connections between indicators (factors). According to our best knowledge, no paper about the application of this method in finance currently exists.

International studies have examined the problem of the impact of macroeconomic factors on stock market changes for many years. In particular, these studies involved well-developed capital markets: United States, Japan, Great Britain, Germany and so on. They usually concentrated on key determinants like inflation, interest rate, industrial output, GDP growth, crude oil prices, money supply, exchange rate or consumption. Nelson (1976) analysed the impact of the inflation rate (both expected and unexpected) on stock market prices in the American market and indicated that inflation growth negatively influences the whole market. Similar conclusions can also be found in (Humpe, Macmillan, 2009). The authors additionally showed a positive impact of the industrial output both on the American and Japanese market. Chen, Roll and Ross (1986) and Fama (1981) presented similar findings. Studies show that the largest influence from macroeconomic data is typical for the GDP, and the GDP growth stimulates upward trends in stock markets. Such conclusions come from, among others, Fama (1981) and McMillan (2010). A positive impact of the exchange rate was shown in the Canadian market research (Dadgostar, Moazzami, 2003) and the Japanese study (Mukherjee, Naka, 1995). A comprehensive review of the aforementioned research, including the direction of the relation between economic factors and the market behaviour was presented, among others, in the doctoral thesis by Wiśniewski (2014).
2. Divisia index approach and its extended version

A key element of the Divisia index approach was to introduce time as a continuous variable $t$ (Divisia, 1925). In the original work, Divisia considers two deterministic functions connected with prices and quantities of the considered commodities (factors $P(t)$ and $Q(t)$ respectively) and he assumes that these continuous functions exist at any point in time. Thus, the value function for the considered set of commodities can be expressed as

$$V(t) = P(t)Q(t).$$

(1)

The next step in Divisia’s approach was to consider differential changes of the value function and that leads to the following formula:

$$dV(t) = Q(t)dP(t) + P(t)dQ(t)$$

(2)
or equivalently:

$$\hat{\delta}^{\text{REG}} = \sum_{i \in s} d_i y_i + \left( \sum_{i \in \Omega} x_i - \sum_{i \in s} d_i x_i \right) \hat{B}$$

(3)

where $L$ denotes the curve of the factors’ change.

The above-mentioned publication suggests consideration of the two factors (variables) as continuous functions of time $t$. It can be easily generalized to a case with $n$ factors influencing the value of $V$ (time notation is omitted for convenience), namely when

$$V = X_1X_2...X_n,$$

(4)

where

$$\Delta V = \int_L X_2X_3...X_n dX_1 + \int_L X_1X_3...X_n dX_2 + ... + \int_L X_1X_2...X_{n-1}dX_n.$$

(5)

Divisia’s idea was developed by Sheremet et al. (1971) and Vaninsky (1987). These publications extended the factorial decomposition described in (4) and (5) to the case with continuously differential function of factors, i.e.

$$V = f(X_1X_2...X_n),$$

(6)

with

$$\Delta V = \int_L \nabla V \circ dX,$$

(7)
where $\nabla V = [f_1, f_2, \ldots, f_n]$ is a gradient vector of the function $f(X_1, X_2, \ldots, X_n)$, $dX = (dX_1, dX_2, \ldots, dX_n)$ and the symbol “$\cdot$” denotes the inner product of two vectors. Vaninsky and Meerovich (1978) extended the case described in (6) and (7) by adding equations of the factors’ interconnection

$$\varphi_j(X_1, X_2, \ldots, X_n) = 0, \text{ for } j = 1, 2, \ldots, m. \quad (8)$$

As a result, the following formula was obtained

$$\Delta V = \int_L \nabla V(I - \Phi \Phi^+) \cdot dX, \quad (9)$$

where $I$ denotes the $n \times n$ identity matrix, $\Phi$ is a Jacobian matrix described as follows

$$\Phi_{ij} = \frac{\partial \Phi_j}{\partial X_i}, \ i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m, \quad (10)$$

and the upper index “$+$” denotes the generalized inverse matrix. If the columns of the matrix $\Phi$ are linearly independent, then we obtain (Vaninsky, 2013):

$$\Phi^+ = (\Phi^T \Phi)^{-1} \Phi^T. \quad (11)$$

It is worth adding that index decompositions based on Divisia’s idea concern both discrete and continuous time and can be found in the following sample papers: Ang, Liu, Chew (2003), Ang (2004), Choi and Oh (2014).

### 3. Choice of variables

Referring to previous studies from well-developed capital markets, we chose initial determinants of the return from stock market indexes in Poland. GDB growth (GDP), industry production, oil prices, money supply, exchange rates, consumption expenditure, indices of international stock exchanges were classified as stimulating factors. In turn, inflation (CPI), interest rate (IR) and unemployment (UNEMPL) should have negative impacts on returns form stock indices. Initial selection, which was aimed at reducing the number of explanatory variables in the model, excluded the industry production, oil prices, money supply, consumption expenditure and interest rate from further consideration. The main argument for rejecting these indicators was the redundancy of part of the information. Since the majority of the
Polish international trade is settled in euro, the EUR/PLN rate was chosen as the exchange rate. The American stock exchange index S&P500, having the strongest correlation with the Polish WIG, was set as the foreign market indicator.

We estimated correlations between the WIG index returns and chosen indicators in the whole time interval. First we checked if the time series were stationary, and in the case of WIG, EUR/PLN, GDP, CPI, UNEMPL the processes were transformed into stationary processes. In all cases the results were in line with the assumed direction of impact on the WIG. The largest, in absolute terms, positive impact was observed for the S&P500 (Table 1) and for the GDP and CPI while the UNEMPL and EUR/PLN exhibited negative correlation.

Table 1. Correlation coefficients between WIG index returns and selected variables in the whole period

<table>
<thead>
<tr>
<th>EUR/PLN</th>
<th>GDP</th>
<th>CPI</th>
<th>UNEMPL</th>
<th>S&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.3632</td>
<td>0.2949</td>
<td>0.0002</td>
<td>-0.0228</td>
<td>0.7026</td>
</tr>
</tbody>
</table>

Source: own work

The CPI index was rejected from further study due to the correlation coefficient being very close to zero for the whole period.

Figure 1 presents the WIG index and the other main indicators which were selected to show the influence of economic factors on the Polish capital market.
The figure suggests that there is a strong dependence between the considered factors and the WIG index. In the subsequent study we examined the influence of economic factors with a lag of 1, 2 or 3 quarters. The results showed that the WIG responds mainly to GDP changes with a lag of 3 quarters (the largest value of the correlation coefficient), however the differences are subtle. For other variables, such a shift was not observed.

4. Factorial decomposition of changes in Warsaw Stock Exchange index fluctuations

In the empirical part of this paper we refer to the generalized Divisia index method (GDIM) with interconnected factors described in (8)–(11). Many authors apply the presented approach or its discrete version to the factorial decomposition of CO\(_2\) emission (Fernández González, Presno, Landajo, 2015; Vaninsky, 2013), changes in the production level (Vaninsky, 1986) or changes in energy intensity (Choi, Oh, 2014). According to our best knowledge, there are no financial applications of the discussed methodology. We verify the applicability of the GDIM to financial market research by examining fluctuations of the Warsaw Stock Exchange WIG Index (WIG). We consider four main factors of WIG changes: \(X_1 = PLN/EUR\) rate, \(X_3 = GDP\) growth, \(X_5 = S&P500\) and the unemployment rate (\(UNEMPL\)) transformed into a the-bigger-the-better variable, i.e. \(X_7 = 1 – UNEMPL\). Let us also denote by \(V = WIG\) and we add four artificial variables: \(X_2 = WIG/(PLN/EUR)\), \(X_4 = WIG/GDP\) growth, \(X_6 = WIG/S&P500\) and \(X_8 = WIG/(1 – UNEMPL)\). Thus we can write:

\[
V = X_1X_2 = X_3X_4 = X_5X_6 = X_7X_8.
\] (12)

In other words we have \(V = X_1X_2\) with the factors’ interconnections written as follows:

\[
\phi_1(X_1,\ldots,X_8) = X_1X_2 - X_3X_4 = 0,
\] (13)

\[
\phi_2(X_1,\ldots,X_8) = X_3X_4 - X_5X_6 = 0,
\] (14)

\[
\phi_3(X_1,\ldots,X_8) = X_5X_6 - X_7X_8 = 0.
\] (15)
The form of the function $V$ and relations (13)–(15) allow us to write:

$$\nabla Z = [X_2, X_1, 0, 0, 0, 0, 0]$$  \hspace{1cm} (16)

and

$$\mathbf{\Phi} = \begin{pmatrix} X_2 & 0 & 0 \\ X_1 & 0 & 0 \\ -X_4 & X_4 & 0 \\ -X_3 & X_3 & 0 \\ 0 & -X_6 & X_6 \\ 0 & -X_5 & X_5 \\ 0 & 0 & -X_8 \\ 0 & 0 & -X_7 \end{pmatrix}.$$  \hspace{1cm} (17)

As a consequence we obtain:

$$\Delta V = \int_{L} \nabla V (1 - \Phi \Phi^+) \circ dX = \sum_{k=1}^{8} \int_{L} \Delta V_{X_k} dX_k,$$  \hspace{1cm} (18)

where:

$$\nabla V_{x_1} = X_2 (X_3^2 + X_4^2) (X_5^2 + X_6^2) (X_7^2 + X_8^2) / D,$$  \hspace{1cm} (19)
$$\nabla V_{x_2} = X_1 (X_3^2 + X_4^2) (X_5^2 + X_6^2) (X_7^2 + X_8^2) / D,$$  \hspace{1cm} (20)
$$\nabla V_{x_3} = X_4 (X_1^2 + X_2^2) (X_5^2 + X_6^2) (X_7^2 + X_8^2) / D,$$  \hspace{1cm} (21)
$$\nabla V_{x_4} = X_3 (X_1^2 + X_2^2) (X_5^2 + X_6^2) (X_7^2 + X_8^2) / D,$$  \hspace{1cm} (22)
$$\nabla V_{x_5} = X_6 (X_1^2 + X_2^2) (X_3^2 + X_4^2) (X_7^2 + X_8^2) / D,$$  \hspace{1cm} (23)
$$\nabla V_{x_6} = X_5 (X_1^2 + X_2^2) (X_3^2 + X_4^2) (X_7^2 + X_8^2) / D,$$  \hspace{1cm} (24)
$$\nabla V_{x_7} = X_8 (X_1^2 + X_2^2) (X_3^2 + X_4^2) (X_5^2 + X_6^2) / D,$$  \hspace{1cm} (25)
$$\nabla V_{x_8} = X_7 (X_1^2 + X_2^2) (X_3^2 + X_4^2) (X_5^2 + X_6^2) / D.$$  \hspace{1cm} (26)
with

\[ D = (X_3^2 + X_4^2)(X_5^2 + X_6^2)(X_7^2 + X_8^2) + \\
+ X_1^2 ((X_5^2 + X_6^2)(X_7^2 + X_8^2) + X_3^2 (X_5^2 + X_6^2 + X_7^2 + X_8^2) + \\
+ X_4^2 (X_5^2 + X_6^2 + X_7^2 + X_8^2)) + X_2^2 ((X_5^2 + X_6^2)(X_7^2 + X_8^2) + \\
+ X_3^2 (X_5^2 + X_6^2 + X_7^2 + X_8^2) + X_4^2 (X_5^2 + X_6^2 + X_7^2 + X_8^2)). \] (27)

In order to make calculations, we need to parameterize the curve of the factors’ dynamics. It is typical for the analytical purposes to assume a linear or an exponential change in the quantitative indicators in time (Vaninsky, 2013) and it seems to be justified by relatively short time subintervals (here quarters) considered in our research. For the purpose of illustration, we assume linear changes of indicators in each quarter. In particular, we use a model time \( t \) that varies in the time interval \([0, 1]\) for each quarter. It may be shown that the length of the time interval does not affect the final results. We assume the WIG, PLN/EUR, GDP and UNEMPL have linear dynamics (between quarters), i.e.:

\[ V(t) = V(0) + (V(1) − V(0)) \cdot t, \] (28)

\[ X_k(t) = X_k(0) + (X_k(1) − X_k(0)) \cdot t, \quad k = 1, 3, 5, 7, \] (29)

where:

\[ dV = V(1) − V(0), \] (30)

\[ dX_k = (X_k(1) − X_k(0)). \] (31)

The results presented in Section 4 are obtained in terms of the relative change in the value of Warsaw Stock Exchange Index (WIG). All calculations were made in Mathematica 6.0.

5. Empirical study

5.1. Data set description and characteristics of time intervals

In this section we apply the discussed approach to examine fluctuations of the Warsaw Stock Exchange WIG index (WIG). As it was mentioned before, we take four main factors of WIG changes into consideration: \( X_1 = PLN/EUR \) rate, \( X_3 = GDP \),
$X_5 = S&P500$, the unemployment rate ($UNEMPL$) transformed into a the-bigger-the-better variable, i.e. $X_7 = 1 – UNEMPL$ and some other, artificial variables presented in Section 3.

As a representative of the Polish capital market, we used the broadest stock market index (WIG), which offers the most complete description of the condition of the Polish economy. We conducted the study for the period from the 2nd quarter of 2003 (Q1 2003) to the 3rd quarter of 2014 (Q3 2014), which was divided into 5 subperiods characterised by subsequent downward and upward trends. As a result we were able to study various market conditions. The choice of such a long study period (over 10 years) guarantees that the data is neither specific nor unrepresentative. The period includes the bull market connected with Polish accession to the European Union as well as the time of the global financial crisis. Figure 2 presents the whole study period and its subperiods.

![Figure 2. WIG index between Q1 2003 and Q3 2014](source: own work)

The chosen periods have various lengths (from 4 to 18 quarters) and substantial differences regarding returns. In the whole study period the WIG index growth amounted to 292.6%. This was the result of upward trends in the bull-market periods (respectively 331.88%, 102.69% and 32.98%) and downward movements (respectively –63.71% and –15.1%). Relevant data is presented in detail in Table 2. The choice of such subperiods allowed us to check the performance of our model in various market conditions.
Table 2. Rate of returns of WIG Index in selected periods

<table>
<thead>
<tr>
<th>Period No.</th>
<th>The initial quarter</th>
<th>The final quarter</th>
<th>% WIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Q1 2003</td>
<td>Q2 2007</td>
<td>372.72</td>
</tr>
<tr>
<td>2</td>
<td>Q2 2007</td>
<td>Q1 2009</td>
<td>−63.71</td>
</tr>
<tr>
<td>3</td>
<td>Q1 2009</td>
<td>Q1 2011</td>
<td>102.69</td>
</tr>
<tr>
<td>4</td>
<td>Q1 2011</td>
<td>Q4 2011</td>
<td>−15.10</td>
</tr>
<tr>
<td>5</td>
<td>Q4 2011</td>
<td>Q3 2014</td>
<td>32.98</td>
</tr>
<tr>
<td>The whole interval</td>
<td>Q1 2003</td>
<td>Q3 2014</td>
<td>292.60</td>
</tr>
</tbody>
</table>

Source: own work

Quarterly data was used for all considered variables, mainly due to the availability of the most important factor determining changes in the stock market index, which is the GDP growth. Other indicators were transformed into quarterly data (PLN/EUR, S&P500) or their final values for relevant quarters were used (UNEMPL).

5.2. Results

We investigated to what extent the changes in the PLN/EUR, GDP, S&P500 and unemployment rate affected the WIG value increase (or decrease). For each time interval we measured the contribution of the considered variables to the relative change in WIG value (see Table 3) by using the following formula: \( \int_{L}^{\Delta V_{X_{i}}} dX_{k} \) (see (18)–(27)). Our results are presented in Table 3.

Table 3. Factors’ contribution to the change in WIG value depending on time interval

<table>
<thead>
<tr>
<th>Factors</th>
<th>Contribution to the relative change in WIG value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period 1</td>
</tr>
<tr>
<td>( X_{1} )</td>
<td>3.948</td>
</tr>
<tr>
<td>( X_{2} )</td>
<td>77.289</td>
</tr>
<tr>
<td>( X_{3} )</td>
<td>79.469</td>
</tr>
<tr>
<td>( X_{4} )</td>
<td>9.913</td>
</tr>
<tr>
<td>( X_{5} )</td>
<td>45.621</td>
</tr>
<tr>
<td>( X_{6} )</td>
<td>78.921</td>
</tr>
<tr>
<td>( X_{7} )</td>
<td>8.512</td>
</tr>
<tr>
<td>( X_{8} )</td>
<td>79.050</td>
</tr>
<tr>
<td>Total percentage change</td>
<td>372.720</td>
</tr>
</tbody>
</table>

Source: own work

We can observe that as a rule (especially during the market prosperity) GDP and S&P500 movements exhibit the strongest influence on the WIG changes. The
PLN/EUR value and the unemployment rate seem to have marginal influence on the WIG fluctuations, only 2.6% and 9%, respectively. Also, in all sub-periods, the influence of PLN/EUR changes seems to be minor. In the first two periods of the bear market and the first two periods of the bull market, the EUR/PLN strengthened the WIG index trend. Only in the period 5 did the EUR/PLN changes result in slowing down the index growth. The level of the unemployment rate positively influenced the growth of the index, but in sub-periods 2 and 3 changes in the unemployment rates adversely affected changes of the WIG index.

The studies showed that the GDP growth and the S&P500 index had significant impacts on the change of the WIG index over the whole period. In sub-periods 1–3 the GDP influenced the WIG strengthening its trend. In the sub-period 4 the GDP slowed down the downward trend. In turn, in the sub-period 5 there was no impact on the index.

The American index had the greatest impact on the level of the WIG index. Movements in the Warsaw Stock Exchange followed the trends in the US. These results confirm a very high correlation between the Polish and the American capital markets. In addition, in all sub-periods the changes in the Polish market were consistent with changes in the US market. As a consequence the US market strengthened the observed trends.

6. Conclusions

This paper presents a tool for factorial decomposition of WIG changes by interconnected factors: the GDP growth, PLN/EUR rate, S&P500 index, and unemployment rate. Due to computational reasons we applied the transformation that produces variables in the form the bigger the better. For analytical purposes only, we also used some artificial factors (like the WIG/GDP or WIG/S&P500), which have poor interpretation. As it was mentioned above, our main conclusion is that GDP and S&P500 fluctuations have the strongest influence on WIG changes. Another observation is that drops in the S&P500 index may have the result that, in spite of positive signals from the economy, the WIG index also goes down. This means that international trends and global investors’ decisions have the largest influence on market behavior in the Warsaw Stock Exchange and fundamental factors from the Polish market are of secondary importance.

The presented method cannot serve as a prognostic tool, however it may be helpful in initial variable selection in econometric regression models. Its drawback is computational complexity. On the other hand its main advantage is the possibility to include interconnections between the factors, which are not necessarily linear. Further studies, which would give more detailed results, require a non-linear approximation of the considered processes in quarterly time intervals and in-
clusion of a higher frequency of the data. There are also some limitations of the presented method in the area of interpretation, i.e. we do not have any information indicating whether the considered interconnected factors are statistically significant. All considered processes have a deterministic character and must be approximated by continuous functions. Since the model is deterministic we cannot use the inferential statistics for the WIG change description and, in particular, we cannot generalize conclusions. However, after the application of the Divisia method, we obtain the knowledge about the factor influence on the dependent variable changes in the considered time interval. It is worth adding that the stochastic generalization of the Divisia price index can be found in literature (Białek, 2015) but, according to our best knowledge, there are no stochastic generalizations for the extended Divisia price index with interconnected factors. Finally, we would like to stress that the aim of the paper is to present some interesting method with interconnected factors rather than to evaluate it in competition with existing approaches. This paper does not play the role of a method proposition since the method is already known and thus we do not compare it to other existing methods of WIG changes analysis, like econometric ones. As a rule, Divisia’s approach is treated as some theoretically perfect method without practical applications (von der Lippe, 2007). We merely hope to change this opinion by showing its possible application for WIG fluctuations analysis.

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Zastosowanie indeksu Divisia z powiązanymi czynnikami do analizy fluktuacji indeksu WIG


Słowa kluczowe: analiza czynnikowa, indeks Divisia, czynniki powiązane, indeks WIG

JEL: C43, G10