Folia Oeconomica
Acta Universitatis Lodziensis
ISSN 0208-6018 e-ISSN 2353-7663

www.czasopisma.uni.lodz.pl/foe/
5(356) 2021
DOI: https://doi.org/10.18778/0208-6018.356.01

Małgorzata Graczyk https://orcid.org/0000-0002-5550-7007

Poznań University of Life Sciences, Faculty of Agronomy and Bioengineering
Department of Mathematical and Statistical Methods, Poznań, Poland
malgorzata.graczyk@up.poznan.pl

Bronisław Ceranka https://orcid.org/0000-0002-5395-1593

Poznań University of Life Sciences, Faculty of Agronomy and Bioengineering
Department of Mathematical and Statistical Methods, Poznań, Poland
bronislaw.ceranka@up.poznan.pl

Notes on Selected Optimal Weighing Designs

Abstract: In this paper, some problems related to the construction of highly D-efficient spring balance weighing designs are presented. We give some conditions determining the relations between the parameters of such designs and construction examples.

Keywords: D-efficient design, spring balance weighing design

JEL: C02, C18, C90

1. Introduction

In this study we consider the class $\boldsymbol{\Psi}_{n\times p}(0,1)$ of $n\times p$ matrices $\mathbf{X}=(x_{ij})$, $i=1,2,\ldots,n$, $j=1,2,\ldots,p$. Any matrix $\mathbf{X}$ belonging to the class $\boldsymbol{\Psi}_{n\times p}(0,1)$ is called the design matrix of the spring balance weighing design. In the first papers on weighing designs, spring balance weighing design was used for determining unknown weights of objects by use of balance with one pan, i.e. spring balance. Now, the applications of mentioned designs are unlimited and connected with economic survey (Banerjee, 1975; Ceranka, Graczyk, 2014), with agricultural experiments (Ceranka, Katulska, 1987a; 1987b; 1989; Graczyk, 2013) or with bioengineering (Gawande, Patkar, 1999). Several issues presented in the literature concerned on weighing designs are linked with optimality criteria (Jacroux, Notz, 1983; Koukouvinos, 1996) and with new construction methods (Gail, Kiefer, 1982; Ceranka, Graczyk, 2010; 2012; Katulska, Smaga, 2010).

Here we study linear model $\mathbf{y}=\mathbf{X}\mathbf{w}+\mathbf{e}$, where $\mathbf{y}$ is an $n\times 1$ random vector of observed measurements, $\mathbf{X}\in\boldsymbol{\Psi}_{n\times p}(0,1)$, $\mathbf{w}$ is a $p\times 1$ vector representing unknown measurements of objects and e is an $n\times 1$ vector of random errors. Moreover, we assume that there are no systematic errors, the errors are uncorrelated and they have different variances, i.e. E($\mathbf{e}$) = $\mathbf{0}_n$, Var($\mathbf{e}$) = $\sigma^2\mathbf{G}$, where $\sigma > 0$ is known parameter, $\mathbf{G}$ is the $n\times n$ diagonal positive definite matrix of known elements.

For the estimation of the vector of unknown measurements of objects $\mathbf{w}$, we use the normal equation $\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\hat{\mathbf{w}}=\mathbf{X}'\mathbf{G}^{-1}\mathbf{y}$. Owing to the fact that G is known positive definite matrix, $\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}$ is nonsingular if and only if X is of full column rank. In that case $\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}$ is nonsingular and the generalized least squares estimator of w is given by $\hat{\mathbf{w}}=(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{G}^{-1}\mathbf{y}$ and Var($\hat{\mathbf{w}}$) = $\sigma^2(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X})^{-1}$.

In this paper, D-optimal designs are considered. The concept of D-optimality was presented in Raghavarao (1971), Banerjee (1975), Shah and Sinh (1989). The design $\mathbf{X}_{\mathrm{D}}$ is regular D-optimal in the class $\boldsymbol{\Psi}_{n\times p}(0,1)$ if

$$\det\left(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\right)^{-1}=\min\left\{\det\left(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\right)^{-1}:\mathbf{X}\in\boldsymbol{\Psi}_{n\times p}(0,1)\right\}.$$

For any number of objects and measurements we are not able to determine regular D-optimal design. In such a case, a highly D-efficient design is considered (Bulutoglu, Ryan, 2009; Ceranka, Graczyk, 2018; Graczyk, Ceranka, 2019). D-efficiency is defined as

$$\mathrm{D}_{\mathrm{eff}}=\left[\frac{\det\left(\mathbf{X}'\mathbf{X}\right)}{\det\left(\mathbf{Y}'\mathbf{Y}\right)}\right]^{1/p},$$

where $\mathbf{Y}$ is the matrix of D-optimal spring balance weighing design. We indicate the highly D-efficient design when $\mathrm{D}_{\mathrm{eff}} \geq 0.95$.

The aim of this paper is to present new results related to the construction methods and optimality existence conditions linked to the D-optimal and highly D-efficient spring balance weighing designs under assumption that the random errors are uncorrelated and they have different variances.

2. The main result

We remind the theorem determining the parameters of the highly D-efficient design given in Ceranka and Graczyk (2018), Graczyk and Ceranka (2019).

Let p be even. In any non-singular spring balance weighing design $\mathbf{X} \in \mathbf{\Psi}_{n\times p}(0,1)$ having $0.5p$ ones in each row and with the variance matrix of errors $\sigma^2\mathbf{I}_n$, $\det(\mathbf{X'X}) \leq (p-1)\left(\frac{np}{4(p-1)}\right)^p$. An upper bound is attained if and only if $\mathbf{X'X} = \frac{n}{4(p-1)}\left(p\mathbf{I}_p + (p-2)\mathbf{1}_p\mathbf{1}_p'\right)$, where $0.25n(p-2)(p-1)^{-1}$ and $0.25np(p-1)^{-1}$ are integers.

The design $\mathbf{X} \in \mathbf{\Psi}_{n\times p}(0,1)$ having form given in Theorem 2.1 is considered as highly D-efficient. Graczyk and Ceranka (2019) gave the issues linked to the addition of one, two and three measurements. Now, we focus on the problem of adding four measurements to the design matrix of the highly D-efficient spring balance weighing design.

Let $\mathbf{X}_1 \in \mathbf{\Psi}_{(n-4)\times p}(0,1)$ be the design of the highly D-efficient spring balance weighing design. Now, let us consider the design $\mathbf{X} \in \mathbf{\Psi}_{n\times p}(0,1)$ in the form:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{x}_1' \\ \mathbf{x}_2' \\ \mathbf{x}_3' \\ \mathbf{x}_4' \end{bmatrix}, \tag{1}$$

where $\mathbf{x}_h'\mathbf{1}_p = t_h$, $\mathbf{x}_h'\mathbf{x}_s = u_{hs}$, $h,s = 1,2,3, h \neq s$. The variance matrix of errors is given as:

$$\mathbf{G}=\begin{bmatrix} \mathbf{I}_{n-4} & \mathbf{0}_{n-4} & \mathbf{0}_{n-4} & \mathbf{0}_{n-4} & \mathbf{0}_{n-4} \\ \mathbf{0}'_{n-4} & g_1^{-1} & 0 & 0 & 0 \\ \mathbf{0}'_{n-4} & 0 & g_2^{-1} & 0 & 0 \\ \mathbf{0}'_{n-4} & 0 & 0 & g_3^{-1} & 0 \\ \mathbf{0}'_{n-4} & 0 & 0 & 0 & g_4^{-1} \end{bmatrix}. \quad (2)$$

In order to study the properties of function $\det\left(\mathbf{X'G}^{-1}\mathbf{X}\right)$ we consider $\mathbf{X'}_1\mathbf{X}_1=\frac{n-4}{4(p-1)}\left(p\mathbf{I}_p+(p-2)\mathbf{1}_p\mathbf{1}'_p\right)$. So, $\det\left(\mathbf{X'}_1\mathbf{X}_1\right)\le(p-1)\left(\frac{(n-4)p}{4(p-1)}\right)^p$ and the upper bound is attained if and only if $\frac{(n-4)p}{4(p-1)}$ and $\frac{(n-4)(p-2)}{4(p-1)}$ are integer. Consequently

$$\det\left(\mathbf{X'G}^{-1}\mathbf{X}\right)\le(p-1)\left(\frac{(n-4)p}{4(p-1)}\right)^p\det\left(\mathbf{I}_4+\begin{bmatrix} g_1\mathbf{x}'_1 \\ g_2\mathbf{x}'_2 \\ g_2\mathbf{x}'_3 \\ g_2\mathbf{x}'_4 \end{bmatrix}\left(\mathbf{X'}_1\mathbf{X}_1\right)^{-1}\left[\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{x}_3\ \mathbf{x}_4\right]\right).$$

Since $\left(\mathbf{X}'_1\mathbf{X}_1\right)^{-1}=\frac{4(p-1)}{(n-4)p}\left(\mathbf{I}_p-\frac{p-2}{p(p-1)}\mathbf{1}_p\mathbf{1}'_p\right)$, then

$$\det\left(\mathbf{X'G}^{-1}\mathbf{X}\right)\le(p-1)\left(\frac{(n-4)p}{4(p-1)}\right)^p\det(\mathbf{M}),$$

where

$$\mathbf{M}=\mathbf{I}_4+\begin{bmatrix} g_1\mathbf{x}'_1 \\ g_2\mathbf{x}'_2 \\ g_2\mathbf{x}'_3 \\ g_2\mathbf{x}'_4 \end{bmatrix}\frac{4(p-1)}{(n-4)p}\left(\mathbf{I}_p-\frac{p-2}{p(p-1)}\mathbf{1}_p\mathbf{1}'_p\right)\left[\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{x}_3\ \mathbf{x}_4\right].$$

The diagonal elements of **M** are equal $m_{hh}=1+\eta g_h\left(t_h-\psi t_h^2\right)$ and off-diagonal elements are equal $m_{hs}=1+\eta g_h\left(u_{hh'}-\psi t_h t_{h'}\right)$, $\eta=\frac{4(p-1)}{(n-4)p}$, $\psi=\frac{p-2}{p(p-1)}$, $h,s=1,2,3,4, h\ne s$.

By extension we obtain $\det(\mathbf{M}) = \mathrm{A} + 2\eta^3\mathrm{B} - \eta^4(\mathrm{F}+\mathrm{G})$, where

$$\mathrm{A} = \prod_{h=1}^{4}\left(1+\eta g_h\left(t_h - \psi t_h^2\right)\right),\ \mathrm{B} = \prod_b g_b \prod_s \left(u_{bs} - \psi t_b t_s\right)\sum_{h=1}^{4}\left(1+\eta g_h\left(t_h - \psi t_h^2\right)\right),$$

$$\mathrm{F} = \prod_{h=1}^{4} g_h\left(u_{13} - \psi t_1 t_3\right)\left(u_{24} - \psi t_2 t_4\right)\sum_{s=1}^{2}\left(u_{s,s+1} - \psi t_s t_{s+1}\right)\left(u_{s+2,s+3} - \psi t_{s+2} t_{s+3}\right),$$

$$\begin{gathered}\mathrm{G} = (\prod_{h=1}^{4} g_h\left(u_{12} - \psi t_1 t_2\right)\left(u_{24} - \psi t_2 t_4\right) + \prod_{h=1}^{4} g_h\left(u_{14} - \psi t_1 t_4\right) \\ \left(u_{23} - \psi t_2 t_3\right))\sum_{s=1}^{2}\left(u_{s+1,s+2} - \psi t_{s+1} t_{s+2}\right)\left(u_{s+2,s+3} - \psi t_{s+2} t_{s+3}\right)\end{gathered},$$

$h,b,s = 1,2,3,4, h \neq b \neq s$. Because $t_h - \psi t_h^2 \leq \frac{p^3+8}{4p(p-1)}$, then the equality is fulfilled if and only if $t_h = \frac{p+2}{2}, h = 1,2,3,4$.

In order to maximise $\det(\mathbf{T})$, we determine the maximum value of integer number u_{hs}. Now, we consider two cases: $p \equiv 0(mod\,4)$ and $p+2 \equiv 0(mod\,4)$.
If $p = 0(mod4)$ then the minimal value of $u_{hs} - \psi t_h t_s$ equals $\frac{p^2+8}{4p(p-1)}$ for $u_{hs} = 0.25(p+4)$. In this case:

$$\det(\mathbf{M}) \leq K + 2\eta^3 L - 3\nu^4 M - \nu^2 N, \tag{3}$$

where

$$K = \prod_{h=1}^{4}\left(1+\varphi g_h\right),\ L = \sum_{h=1}^{4}\prod_{h' \neq h} g_{h'}\left(1+\varphi g_h\right),\ M = \prod_{h=1}^{4} g_h,$$

$$N = \sum_{h=1}^{4} g_h g_{h'}\left(1+\varphi g_q\right)\left(1+\varphi g_b\right),\ \varphi = \frac{p^3+8}{(n-4)p^2},\ \nu = \frac{p^2+8}{(n-4)p^2},$$

$$h,b,s,q = 1,2,3,4, h \neq b \neq s \neq q.$$

If $p+2 = 0(mod4)$ then the minimal value of $u_{hs} - \frac{(p-2)(p+2)^2}{4p(p-1)}$ equals $-\frac{(p+2)(p-4)}{4p(p-1)}$ for $u_{hs} = 0.25(p+2)$. In this case:

$$\det(\mathbf{M}) \leq K - 2\varepsilon^3 L - 3\varepsilon^4 M - \varepsilon^2 N, \tag{4}$$

where, $\varepsilon = \frac{(p+2)(p-4)}{(n-4)p^2}$, $h,b,s,q=1,2,3,4, h \neq b \neq s \neq q$. So, we can formulate the following theorem.

Theorem 2.1.

Any spring balance weighing design $\mathbf{X} \in \boldsymbol{\Psi}_{n \times p}(0,1)$ in the form (1) with the variance matrix of errors in the form (2) is highly D-efficient in the class $\boldsymbol{\Psi}_{n \times p}(0,1)$ if and only if $t_h = \frac{p+2}{2}$, $\frac{(n-4)p}{4(p-1)}$ and $\frac{(n-4)(p-2)}{4(p-1)}$ are integer numbers and:

a) $u_{hs} = 0.25(p+4)$ if $p \equiv 0 \, mod \, 4$,
b) $u_{hs} = 0.25(p+2)$ if $p+2 \equiv 0 \, mod \, 4$, $h,s=1,2,3,4, h \neq s$.

Definition 2.1.

Any spring balance weighing design $\mathbf{X} \in \Psi_{n \times p}(0,1)$ in the form (1) with the variance matrix of errors in the form (2) is highly D-efficient if $\det(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X})$ attains upper bound given in (3) in the case $p \equiv 0 \, mod \, 4$ or attains upper bound given in (4) in the case $p+2 \equiv 0 \, mod \, 4$.

Example 2.1.

Let us consider the variance matrix of errors $\sigma^2\mathbf{G}$, where

$$\mathbf{G}^{-1} = \begin{bmatrix} \mathbf{I}_6 & \mathbf{0}_6 & \mathbf{0}_6 & \mathbf{0}_6 & \mathbf{0}_6 \\ \mathbf{0}_6' & 5 & 0 & 0 & 0 \\ \mathbf{0}_6' & 0 & 3 & 0 & 0 \\ \mathbf{0}_6' & 0 & 0 & 7 & 0 \\ \mathbf{0}_6' & 0 & 0 & 0 & 2 \end{bmatrix}.$$

We determine highly D-efficient design in the class $\mathbf{X} \in \boldsymbol{\Psi}_{10 \times 4}(0,1)$. So, take the highly D-efficient spring balance weighing design $\mathbf{X}_1 \in \boldsymbol{\Psi}_{6 \times 4}(0,1)$ in the form

$$\mathbf{X}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$ If $\mathbf{x}_1' = [1\ 1\ 1\ 0]$, $\mathbf{x}_2' = [1\ 1\ 0\ 1]$, $\mathbf{x}_3' = [1\ 0\ 1\ 1]$, $\mathbf{x}_4' = [0\ 1\ 1\ 1]$ then

$$\mathbf{X} = \begin{bmatrix} & \mathbf{X}_1 & \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \in \boldsymbol{\Psi}_{10\times4}(0,1)$$ is highly D-efficient spring balance weighing design.

Example 2.2.

Let us consider the variance matrix of errors

$\sigma^2\mathbf{G}$, where $\mathbf{G}^{-1} = \begin{bmatrix} \mathbf{I}_{18} & \mathbf{0}_{18} & \mathbf{0}_{18} & \mathbf{0}_{18} & \mathbf{0}_{18} \\ \mathbf{0}_{18}^{'} & 5 & 0 & 0 & 0 \\ \mathbf{0}_{18}^{'} & 0 & 3 & 0 & 0 \\ \mathbf{0}_{18}^{'} & 0 & 0 & 7 & 0 \\ \mathbf{0}_{18}^{'} & 0 & 0 & 0 & 2 \end{bmatrix}$. We determine highly D-efficient design in the class $\mathbf{X} \in \boldsymbol{\Psi}_{22\times10}(0,1)$. So, take the highly D-efficient spring balance weighing design $\mathbf{X}_1 \in \boldsymbol{\Psi}_{18\times10}(0,1)$ in the form

$$\mathbf{X}_1 = \begin{bmatrix} \mathbf{X}_{11} \\ \mathbf{X}_{12} \end{bmatrix}, \mathbf{X}_{11} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix},$$

$$\mathbf{X}_{12} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

If $\mathbf{x}_1^{'} = [1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0]$, $\mathbf{x}_2^{'} = [1\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0]$, $\mathbf{x}_3^{'} = [1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1]$, $\mathbf{x}_4^{'} = [0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1]$, then

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ 1110111000 \\ 1101100110 \\ 1011010101 \\ 0111001011 \end{bmatrix} \in \mathbf{\Psi}_{22\times 10}(0,1)$$

is highly D-efficient spring balance weighing design.

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Uwagi o wybranych optymalnych układach wagowych

Streszczenie: W artykule zostały przedstawione zagadnienia związane z konstrukcją wysoce D-efektywnych sprężynowych układów wagowych. Podane zostały warunki określające zależności pomiędzy parametrami tych układów oraz przykłady konstrukcji.

Słowa kluczowe: D-efektywne układy, sprężynowe układy wagowe

JEL: C02, C18, C90

Received: 2021-11-30; verified: 2021-12-20. Accepted: 2022-01-18