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A PROOF-THEORETIC INTERPOLATION THEOREM FOR INQUISITIVE PROPOSITIONAL LOGIC

Abstract

This paper presents a sequent calculus for Inquisitive Propositional Logic obtained by expanding the sequent calculus g3ip for intuitionistic propositional logic with suitable rules for double negation elimination for atoms and the Split Property. A suitable rule for the Split Property is obtained by taking advantage of the connection between the truth-conditional fragment in Inquisitive Logic and Harrop formulas. The paper proves admissibility of cut for the sequent calculus and uses the sequent calculus to prove interpolation for Inquisitive Propositional Logic. Interpolation is obtained using Maehara's lemma.

Keywords: Inquisitive Logic, Harrop formulas, interpolation, Maehara's lemma, intermediate logics.

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1. Introduction

Inquisitive propositional logic, referred to as basic inquisitive logic (InqB) by [3], is a propositional logic obtained by expanding intuitionistic propositional logic (IPL) with double negation elimination for atoms (DNEp) and the Kreisel-Putnam axiom (KP). The result is a logic for which uniform substitution of arbitrary formulas for atoms does not hold because some formulas will not satisfy double negation elimination. The logic was first presented by [6] and the research programme in its contemporary shape can be found in [4, 3]. The logic is developed to model reasoning involving both statements and questions by interpreting the intuitionistic disjunction $A \vee B$ as "*whether A or* B?".

Natural deduction systems for InqB can be found in [3, 9] and labelled sequent calculi based on the support semantics for InqB can found in [2, 9]. However, there is no simple and standard two-sided sequent calculus for InqB, that is, a calculus obtained by extending a standard sequent calculus for IPL with appropriate introduction rules equivalent to DNEp and KP and for which cut is admissible. In addition, there is no syntactic proof of an interpolation theorem for InqB. In fact, I am not aware of any interpolation theorem for InqB. While there is an interpolation theorem for the variant of inquisitive propositional logic explored in [5], and the "folklore" is that their result transfers to InqB, that remains unpublished. The aim of this paper is to rectify this situation by developing such a sequent calculus for InqB and use it to provide a syntactic proof of interpolation specifically for InqB.

The next section presents InqB and highlights the connection with Harrop formulas. The third section presents the sequent calculus. The fourth section establishes that cut is admissible for the sequent calculus and the fifth section presents the interpolation theorem. The final section concludes by briefly discussing decidability and future work.

2. Towards a sequent calculus for InqB

I will use a standard propositional language with countably many atoms, $\to$, $\wedge$, $\vee$ and $\bot$. Lower case Latin letters starting from p represent atoms, and upper case Latin letters starting from A represent arbitrary formulas. $\neg A$ is defined as $A \to \bot$. As mentioned above, InqB can be obtained by expanding an axiomatisation of IPL with the following axioms:

$$\neg\neg p \to p \tag{DNEp}$$

$$(\neg C \to (A \vee B)) \to ((\neg C \to A) \vee (\neg C \to B)) \tag{KP}$$

The last axiom is the Kreisel-Putnam axiom. Going forward, I will for readability drop the innermost parentheses in both the antecedent and the consequent by writing

$$(\neg C \to A \vee B) \to (\neg C \to A \vee \neg C \to B)$$

This also goes for variations of this axiom and rules based on them. Thus, unless otherwise specified, a formula of the form $C \to A \vee B$ should be read as $C \to (A \vee B)$ while a formula of the form $C \to A \vee C \to B$ should be read as $(C \to A) \vee (C \to B)$.

KP can be replaced with the following axiom where α is a $\vee$-free formula:

$$(\alpha \to A \vee B) \to (\alpha \to A \vee \alpha \to B) \tag{Split$\vee$}$$

They are intersubstitutable because every formula for which DNE holds is equivalent to a $\vee$-free formula and every $\vee$-free formula is equivalent to some negated formula within InqB [3, p. 65].

In IPL, DNE only holds for a formula A if A or $A \to \bot$ is a theorem of IPL. InqB strictly extends the set of formulas for which DNE holds. This fragment is referred to as the truth-conditional fragment [4, 3].

As demonstrated by [12], there is an intimate connection between Harrop formulas and the truth-conditional fragment of the language. A (propositional) Harrop formula as introduced in [7] is defined inductively as follows: atoms and $\bot$ are Harrop formulas; if A and B are Harrop formulas, then $A \wedge B$ is a Harrop formula; if A is a Harrop formula, then for any formula

C, $C \to A$ is a Harrop formula. Harrop formulas were introduced to extend the disjunctive property for IPL from just theorems as follows:

PROPOSITION 2.1 (Harrop [7]). For every Harrop formula $\mathcal{H}$, If $\mathcal{H} \to A \vee B$ is a theorem of IPL, then either $\mathcal{H} \to A$ or $\mathcal{H} \to B$ is a theorem of IPL.

With InqB, this becomes an *object-theoretic* statement in the form of the Split property which then holds for every truth-conditional formula.

Trivially, every negated formula and every $\vee$-free formula is a Harrop formula, and it is shown in [9] that every Harrop formula is truth-conditional. Of course, there are truth-conditional formulas that are not Harrop formulas. This includes not only theorems such as the instances of $A \to (A \vee B)$ but also other formulas such as $(p \to (p \vee q)) \wedge p$. DNE holds for this formula in InqB. That being said, the truth-conditionality of the latter kind of formulas can be accounted for through the notion of a Harrop expansion. In particular, Harrop expansions of truth-conditional formulas are truth-conditional formulas:

- If α and β are truth-conditional, then $\alpha \wedge \beta$ is truth-conditional.
- If α is truth-conditional and C is any formula, then $C \to \alpha$ is truth-conditional.

Establishing whether every truth-conditional formula is either a Harrop formula, a theorem or a Harrop expansion is beyond the scope of this paper. Instead, it suffices for our purposes to observe that Split for Harrop formulas implies Split for $\vee$-free formulas and KP. One can thus define InqB by expanding IPL with DNEp and the following Split axiom for Harrop formulas where $\mathcal{H}$ is a Harrop formula:

$$(\mathcal{H} \to A \vee B) \to (\mathcal{H} \to A \vee \mathcal{H} \to B) \qquad (\text{Split}\mathcal{H})$$

This will turn out to be extremely practical in the following.

3. A sequent calculus for InqB

I present in this section the sequent calculus for InqB together with some basic observations about it. Roughly, the sequent calculus is obtained by expanding the single-succedent sequent calculus g3ip for IPL from [10] with suitable rules to capture DNEp and the Split axiom for Harrop formulas.

DEFINITION 3.1. Let g3InqB be the sequent calculus based on sequents of the form $\Gamma \Rightarrow A$ where Γ is a multiset of formulas with the following initial sequents and rules where A, B and C are arbitrary formulas, p is an atom and $\mathcal{H}$ is a Harrop formula:

$$p, \Gamma \Rightarrow p \qquad \bot, \Gamma \Rightarrow C$$

$$\frac{A, \Gamma \Rightarrow C \qquad B, \Gamma \Rightarrow C}{A \vee B, \Gamma \Rightarrow C}\ \vee\text{L} \qquad \frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\ \vee\text{R}_1 \qquad \frac{\Gamma \Rightarrow B}{\Gamma \Rightarrow A \vee B}\ \vee\text{R}_2$$

$$\frac{A, B, \Gamma \Rightarrow C}{A \wedge B, \Gamma \Rightarrow C}\ \wedge\text{L} \qquad \frac{\Gamma \Rightarrow A \qquad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \wedge B}\ \wedge\text{R}$$

$$\frac{A \to B, \Gamma \Rightarrow A \qquad B, \Gamma \Rightarrow C}{A \to B, \Gamma \Rightarrow C}\ \to\text{L} \qquad \frac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A \to B}\ \to\text{R}$$

$$\frac{\neg p, \Gamma \Rightarrow \bot}{\Gamma \Rightarrow p}\ \text{Raa-at}$$

$$\frac{\mathcal{H} \to A, \Gamma \Rightarrow C \qquad \mathcal{H} \to B, \Gamma \Rightarrow C}{\mathcal{H} \to A \vee B, \Gamma \Rightarrow C}\ \mathcal{H}\textit{Split}\text{L}$$

The sequent calculus g3ip is thus g3InqB without *Raa-at* and $\mathcal{H}$*Split*L. For each application of a rule, I will refer to Γ and C as the context, while the other formula displayed in the conclusion-sequent is the principal formula, and the other formulas displayed in the premise-sequents are the active formulas. The formulas in Γ and C are parametric formulas.

As in the case of g3ip, the following *structural* rules are admissible in g3InqB:

$$\frac{\Gamma \Rightarrow C}{\Gamma', \Gamma \Rightarrow C}\ \text{Weakening} \qquad \frac{A, A, \Gamma \Rightarrow C}{A, \Gamma \Rightarrow C}\ \text{Contraction}$$

$$\frac{\Gamma \Rightarrow A \qquad A, \Gamma' \Rightarrow C}{\Gamma, \Gamma' \Rightarrow C}\ \text{cut}$$

This will be demonstrated below and in the next section.

The rule *Raa-at* is from [10, p. 158]. I am not aware of any previous presentation of the rule $\mathcal{H}$*Split*L (or the corresponding one for KP or Split∨). Instead, one typically finds the following rule or the corresponding one for the ∨-free fragment:

$$\frac{\Gamma, \neg C \Rightarrow (A \vee B)}{\Gamma \Rightarrow \neg C \to A \vee \neg C \to B}\ \text{KP-R}$$

See for example [11] for a natural deduction variant of this rule.

However, this rule is not particularly practical from the perspective of proof analysis and admissibility of cut, a property which is desirable for our purposes since proving interpolation through *Maehara's lemma* requires that atoms occurring in the premise-sequents also occur in the conclusion-sequents. If cut (or a similar rule) must be included as primitive rule in the sequent calculus, then that will not be the case.

Consider for example the following sequent as an instance of Split∨:

$$p \to A \vee B \Rightarrow p \to A \vee p \to B$$

There is no cut-free derivation of that sequent with KP-R. KP-R is thus not suitable for our purposes. Replacing KP-R with the corresponding rule for ∨-free formulas will not solve the issue, since there is in that case no cut-free derivation of for example

$$((p \vee q) \to r) \to (A \vee B) \Rightarrow (((p \vee q) \to r) \to A) \vee (((p \vee q) \to r) \to B)$$

With $\mathcal{H}$*Split*L and the corresponding rule obtained by formulating KP-R for Harrop formulas, this is not an issue.

I suspect that the strategy for establishing admissibility of cut employed below in section 4 will also work with KP-R for Harrop formulas. However, I prefer $\mathcal{H}$*Split*L over the corresponding right introduction rule for various aesthetic reasons. Most importantly, I understand the rule as a structural rule concerning the structure of the antecedent of a sequent: it imitates the use of set-theoretic union of two states in a proof of the validity of (Split$\mathcal{H}$) in the support semantics for InqB. Presenting the support semantics and the proof in question goes beyond the scope of this paper, but the reader may refer to [3] for a presentation of the support semantics. As I see it, a nice hypersequent calculus for InqB would include a structural rule with the appropriate effect.

In this regard, it is also worth comparing $\mathcal{H}$*Split*L with the "higher-level" natural deduction elimination rule $\vee E_+$ for $\vee$ presented by [9, p. 18]. That rule is employed to obtain a normalization theorem for a natural deduction system for InqB. While I will not present the rule itself, it corresponds directly to the following sequent calculus rule:

$$\dfrac{\mathcal{H}, \Gamma \Rightarrow A \vee B \qquad \dfrac{\mathcal{D}[A/\mathcal{H}]}{\Gamma \Rightarrow C} \qquad \dfrac{\mathcal{D}[B/\mathcal{H}]}{\Gamma \Rightarrow C}}{\Gamma \Rightarrow C}\ \mathrm{S}_E$$

The expression $\mathcal{D}[A/\mathcal{H}]$ means that the derivation of the premise-sequent includes one or more applications of a rule of the following form:

$$\dfrac{A, \Gamma \Rightarrow C}{\mathcal{H}, \Gamma \Rightarrow C}$$

It is assumed that the calculus includes one rule of this form for each pair of a Harrop formula and any formula. The formulation of a precise notion of a derivation is beyond the scope of this paper, but the rule S_E must be understood as "discharging" applications of such rules, and the root only counts as derivable if each application of such rules are "discharged" through an appropriate application of S_E.

The rule S_E is currently an elimination rule, but it corresponds to the following introduction rule:

$$\dfrac{\dfrac{}{\Gamma \Rightarrow C}\,[A/\mathcal{H}] \qquad \dfrac{}{\Gamma \Rightarrow C}\,[B/\mathcal{H}]}{\mathcal{H} \to A \vee B, \Gamma \Rightarrow C}\ \mathrm{S}_I$$

The rule $\mathcal{H}$*Split*L is now obtained by replacing the requirement on the derivations with the conditionals $\mathcal{H} \to A$ and $\mathcal{H} \to B$ as active formulas in the premise-sequent. This paper thus demonstrates that the more complicated notion of a derivation which is required to accommodate the "discharging" of applications of $A/\mathcal{H}$-rules is not needed to obtain a sequent calculus for InqB for which cut is admissible and *Maehara's lemma* is applicable.

The rule $\mathcal{H}$*Split*L can also be understood as a simplification of the approach by [1] to a sequent calculus for a related logic which can be described as propositional team logic expanded with inquisitive disjunction. The language in [1] includes two disjunctions, a tensor disjunction and an inquisitive disjunction, where $\vee$ is used for tensor disjunction. I will in my brief discussion continue to use $\vee$ for the inquisitive disjunction. The following *deep inference* rules are presented by [1] where $\Psi(A \vee B)$ is *roughly* a formula in which the subformula $A \vee B$ does not occur within the scope of a negation:

$$\frac{\Psi(A), \Gamma \Rightarrow C \qquad \Psi(B), \Gamma \Rightarrow C}{\Psi(A \vee B), \Gamma \Rightarrow C} \text{Deep-}\vee\text{L} \frac{\Gamma \Rightarrow \Delta, \Psi(A)}{\Gamma \Rightarrow \Delta, \Psi(A \vee B)} \text{Deep-}\vee\text{R}_1$$

Like the rule $\mathcal{H}$*Split*L, the deep inference rules decompose a formula from within. With deep inference rules, one requires only two rules for $\vee$ rather than $\vee$L, $\vee$R and $\mathcal{H}$*Split*L. However, $\mathcal{H}$*Split*L is considerably simpler by only considering a context of a specific form, which in turn simplifies the strategy for establishing admissibility of cut. Specifically, whereas the proof presented below is relatively direct, the proof that cut is admissible in the sequent calculus presented by [1] ends up being considerably more complex because the deep inference rules must be combined with certain syntactic restrictions on the context in other rules.

Finally, I note that the subformula property fails for g3InqB because $\mathcal{H} \to A$ is not a subformula of $\mathcal{H} \to (A \vee B)$ in the rule $\mathcal{H}$*Split*L, and $\neg p$ is not a subformula of p in the rule Raa-at. However, g3InqB still has a *subatom property* in the sense that every atom occurring as a subformula in a leaf of a derivation is a subformula of a formula in the root of that derivation. This holds for g3InqB because there is no rule that removes an atom from the premise-sequents. This property is key for the application of *Maehara's lemma* to establish the interpolation property.

In any case, the admissibility of cut implies the admissibility of modus ponens:

$$\frac{\Gamma \Rightarrow A \to B \qquad \Gamma' \Rightarrow A}{\Gamma, \Gamma' \Rightarrow B}\ \text{MP}$$

With this rule admissible, one can easily show the equivalence of g3InqB and InqB.

PROPOSITION 3.2. If A is a theorem of InqB, then $\Rightarrow A$ is derivable in g3InqB.

PROOF: InqB is IPL expanded with DNEp and KP. KP is subsumed by the Split axiom for Harrop formulas. Since every theorem of IPL is already derivable in g3InqB because the calculus extends g3ip (which is g3InqB without *Raa-at* and $\mathcal{H}$*Split*L), it suffices to show that MP remains admissible, and that the axioms DNEp and Split for Harrop formulas are derivable in g3InqB. Both axioms are obviously derivable, so the desired result follows from theorem 4.2 which implies that MP is admissible. □

PROPOSITION 3.3. If $\Rightarrow A$ is derivable in g3InqB, then A is a theorem of InqB.

PROOF: Since every theorem of IPL is a theorem of InqB, it follows that if $\Rightarrow A$ is derivable in g3ip, then A is a theorem of InqB. It is also the case that the sequent calculus obtained by expanding g3ip with MP, the Split axiom for Harrop formulas and DNEp defines InqB since InqB is IPL expanded with those axioms. It is thus left to show that the rules *Raa-at* and $\mathcal{H}$*Split*L are admissible in that sequent calculus. I illustrate the case of $\mathcal{H}$*Split*L where $\mathcal{H}(D)$ abbreviates $\mathcal{H} \to D$:

$$\dfrac{\Rightarrow \mathcal{H}(A{\vee}B){\to}\, \mathcal{H}(A){\vee}\mathcal{H}(B) \qquad \dfrac{\dfrac{\Gamma, \mathcal{H}(A{\vee}B) \Rightarrow \mathcal{H}(A{\vee}B) \quad \dfrac{\mathcal{H}(A), \Gamma \Rightarrow C \quad \mathcal{H}(B), \Gamma \Rightarrow C}{\mathcal{H}(A){\vee}\mathcal{H}(B), \Gamma \Rightarrow C}\ {\vee}\text{L}}{\mathcal{H}(A{\vee}B), \mathcal{H}(A{\vee}B){\to}\mathcal{H}(A){\vee}\mathcal{H}(B), \Gamma \Rightarrow C}\ {\to}\text{L}}{\mathcal{H}(A{\vee}B), \Gamma \Rightarrow (\mathcal{H}(A{\vee}B){\to}\mathcal{H}(A){\vee}\mathcal{H}(B)){\to}\, C}\ {\to}\text{R}}{\mathcal{H}(A \vee B), \Gamma \Rightarrow C}\ \text{MP}$$

□

I now state some basic lemmas that are required for admissibility of cut. As is usual, the height of a derivation is defined as the longest branch in a derivation. I write $n \vdash \Gamma \Rightarrow C$ for the claim that there is a derivation with height at most n of $\Gamma \Rightarrow C$.

LEMMA 3.4. *The following properties hold in* g3InqB*:*

(a) Height-preserving weakening:

- *if* $n \vdash \Gamma \Rightarrow C$ *then* $n \vdash \Gamma', \Gamma \Rightarrow C$

(b) Height-preserving inversion:

- *For every rule* $\dfrac{\sigma_0 \quad \dots \quad \sigma_m}{\sigma}$ *except* $\rightarrow$*L and* $\vee$*R, if* $n \vdash \sigma$ *then* $n \vdash \sigma_i$ *for* $i \leq m$.
- *For* $\rightarrow$ *L, if* $n \vdash A \rightarrow B, \Gamma \Rightarrow C$ *then* $n \vdash B, \Gamma \Rightarrow C$.

(c) Height-preserving admissibility of contraction:

- *if* $n \vdash A, A, \Gamma \Rightarrow C$ *then* $n \vdash A, \Gamma \Rightarrow C$

PROOF: The proofs are standard, and the reader is referred to [10] for the details with only two exceptions. Inversion for $\mathcal{H}$*Split*L differs slightly from that of $\vee$L, and relies on the inversion of $\vee$L. In addition, one must also consider $\mathcal{H}$*Split*L in the case of inversion for the right premise of $\rightarrow$L.

The proofs proceed by induction on the height of a derivation. In the case of the inductive step for the inversion of $\mathcal{H}$*Split*L I reason as follows. Assume that $n{+}1 \vdash \mathcal{H} \rightarrow A \vee B, \Gamma \Rightarrow C$. If $\mathcal{H} \rightarrow A \vee B$ is not principal, then the inductive hypothesis is applied on the premise-sequent(s) before the rule in question is re-applied. If it is principal, is it obtained with either $\mathcal{H}$*Split*L or $\rightarrow$L. In the former case, the premise-sequents are the desired conclusions themselves. The latter case involves the premise-sequents $\mathcal{H} \rightarrow A \vee B, \Gamma \Rightarrow \mathcal{H}$ and $A \vee B, \Gamma \Rightarrow C$ derivable with height at most n. The inductive hypothesis is applied on the first, and $\vee$L inversion on the second. Applying $\rightarrow$L yields $n{+}1 \vdash \mathcal{H} \rightarrow A, \Gamma \Rightarrow C$ and $n{+}1 \vdash \mathcal{H} \rightarrow B, \Gamma \Rightarrow C$.

Regarding inversion of the right premise of $\rightarrow$L, I reason as follows if $\mathcal{H} \rightarrow A \vee B, \Gamma \Rightarrow C$ is obtained with $\mathcal{H}$*Split*L and $\mathcal{H} \rightarrow A \vee B$ is principal. Then it is obtained from $\mathcal{H} \rightarrow A, \Gamma \Rightarrow C$ and $\mathcal{H} \rightarrow B, \Gamma \Rightarrow C$. I apply the inductive hypothesis and then $\vee$L to obtain the desired result. □

This concludes the preliminary discussion of the sequent calculus. I now proceed to show that cut is admissible in g3InqB by proof analysis.

4. Admissibility of cut

A proof that cut is admissible by proof analysis proceeds typically by induction on the weight of the cut-formula with a subinduction on the cut-height defined as the sum of the heights of the derivations of the premise-sequents. The weight of a complex formula is defined as the sum of the weights of its direct subformulas plus 1. Atoms are assigned 1 and $\bot$ is assigned 0. The cut-formula is the formula displayed in the premise-sequents in the rule. The proof consists in providing transformations that remove the cut (if at least one premise is an initial sequent), or reduce either the weight of the cut-formula or the cut-height.

Consider for example the case where the cut-formula is of the form $A \wedge B$ and it is the principal formula of the rules applied to obtain the premise-sequents:

$$\dfrac{\dfrac{\Gamma \Rightarrow A \qquad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \wedge B}\,\wedge\mathrm{R} \qquad \dfrac{A, B, \Gamma' \Rightarrow C}{A \wedge B, \Gamma' \Rightarrow C}\,\wedge\mathrm{L}}{\Gamma, \Gamma' \Rightarrow C}\,\mathrm{cut}$$

That application of cut is permuted into two applications on lower weight as follows:

$$\dfrac{\dfrac{\Gamma \Rightarrow B \qquad \dfrac{\Gamma \Rightarrow A \qquad A, B, \Gamma' \Rightarrow C}{B, \Gamma, \Gamma' \Rightarrow C}\,\mathrm{cut}}{\Gamma, \Gamma, \Gamma' \Rightarrow C}\,\mathrm{cut}}{\Gamma, \Gamma' \Rightarrow C}\,\mathrm{Contr.}$$

To obtain the desired result, contraction is applied. More details can be found in for example [10].

Whereas the rule *Raa-at* is dealt with in a straight-forward manner, the rule $\mathcal{H}$*Split*L complicates things considerably. Consider the following case:

$$\dfrac{\dfrac{\mathcal{H}, \Gamma \Rightarrow A \vee B}{\Gamma \Rightarrow \mathcal{H} \to A \vee B}\,\to\mathrm{R} \qquad \dfrac{\mathcal{H} \to A, \Gamma' \Rightarrow C \qquad \mathcal{H} \to B, \Gamma' \Rightarrow C}{\mathcal{H} \to A \vee B, \Gamma' \Rightarrow C}\,\mathcal{H}Split\mathrm{L}}{\Gamma, \Gamma' \Rightarrow C}\,\mathrm{cut}$$

As things stand, one cannot apply the inductive hypothesis (i.e. cut) directly on the premise-sequents of $\to$R and $\mathcal{H}Split$L since the active formulas $\mathcal{H} \to A$ and $\mathcal{H} \to B$ in the antecedents of premise-sequents of $\mathcal{H}Split$L are not matched by the active formula $A \vee B$ in the succedent of the premise-sequent of $\to$R. Moreover, applying the inversion lemma on the conclusion-sequent of $\mathcal{H}Split$L will only yield $A \vee B, \Gamma' \Rightarrow C$ which leaves us with $\mathcal{H}, \Gamma, \Gamma' \Rightarrow C$ after cut. Finally, there is obviously no transformation of the derivation of the premise-sequent of $\to$R into $\Gamma \Rightarrow \mathcal{H} \to A, \mathcal{H} \to B$ since the sequents are single-succedents.

However, with $\mathcal{H}$ being a Harrop formula, there is a solution inspired by the strategy employed by [14] to establish admissibility of cut for Gentzen's original sequent calculus without multicut, namely by tracing formulas up through the derivation and applying cut there. In particular, the derivation of $\mathcal{H}, \Gamma \Rightarrow A \vee B$ will have one or more branches with the property that $A \vee B$ is introduced into the succedent position through a leaf or an application of $\vee$R and remains parametric in that position until the root. In each such case where $\mathcal{H}$ isn't already in the context, the introduction of $\mathcal{H}$ can be permuted upwards in the derivation until that is the case. One can then apply cut there before the derivation below is reconstructed. To that purpose, the following lemma is required.

Lemma 4.1. *The rules $\wedge L$ and $\to L$ permute up with respect to every rule in* g3InqB.

Proof: In the case of $\wedge$L, I illustrate the permutation with respect to $\to$L. The others proceed in the same manner.

$$\dfrac{\dfrac{A, B, D \to E, \Gamma \Rightarrow D \qquad A, B, E, \Gamma \Rightarrow C}{A, B, D \to E, \Gamma \Rightarrow C}\to\text{L}}{A \wedge B, D \to E, \Gamma \Rightarrow C}\wedge\text{L}$$

It is permuted into the following:

$$\dfrac{\dfrac{A, B, D \to E, \Gamma \Rightarrow D}{A \wedge B, D \to E, \Gamma \Rightarrow D}\wedge\text{L} \qquad \dfrac{A, B, E, \Gamma \Rightarrow C}{A \wedge B, E, \Gamma \Rightarrow C}\wedge\text{L}}{A \wedge B, D \to E, \Gamma \Rightarrow C}\to\text{L}$$

One simply applies $\wedge$L prior to applying the other rule since the context contains A, B.

In the case of $\to$L, the subcases differ slightly between each other, and inversion is crucial for the permutation up with respect to left introduction rules. Let's consider some subcases.
The case $\to$L below $\vee$L:

$$\dfrac{A \to B, D \vee E, \Gamma \Rightarrow A \qquad \dfrac{B, D, \Gamma \Rightarrow C \qquad B, E, \Gamma \Rightarrow C}{B, D \vee E, \Gamma \Rightarrow C}\,\vee\text{L}}{A \to B, D \vee E, \Gamma \Rightarrow C}\,\to\text{L}$$

It is permuted as follows:

$$\dfrac{\dfrac{\dfrac{A \to B, D \vee E, \Gamma \Rightarrow A}{A \to B, D, \Gamma \Rightarrow A}\,\text{Inv.} \qquad B, D, \Gamma \Rightarrow C}{A \to B, D, \Gamma \Rightarrow C}\,\to\text{L}}{\vdots}$$

$$\dfrac{\dfrac{\vdots}{A \to B, D, \Gamma \Rightarrow C} \qquad \dfrac{\dfrac{A \to B, D \vee E, \Gamma \Rightarrow A}{A \to B, E, \Gamma \Rightarrow A}\,\text{Inv.} \qquad B, E, \Gamma \Rightarrow C}{A \to B, E, \Gamma \Rightarrow C}\,\to\text{L}}{A \to B, D \vee E, \Gamma \Rightarrow C}\,\vee\text{L}$$

The case $\to$L below $\vee$R:

$$\dfrac{A \to B, \Gamma \Rightarrow A \qquad \dfrac{B, \Gamma \Rightarrow D}{B, \Gamma \Rightarrow D \vee E}\,\vee\text{R}}{A \to B, \Gamma \Rightarrow D \vee E}\,\to\text{L}$$

This can be permuted into the following derivation:

$$\dfrac{\dfrac{A \to B, \Gamma \Rightarrow A \qquad B, \Gamma \Rightarrow D}{A \to B, \Gamma \Rightarrow D}\,\to\text{L}}{A \to B, \Gamma \Rightarrow D \vee E}\,\vee\text{R}$$

The case of $\to$L below *Raa-at* proceeds in the same way as $\to$L below $\vee$R.
The case $\to$L $(A \to B)$ below $\to$L $(E \to D)$:

$$\dfrac{A \to B, D \to E, \Gamma \Rightarrow A \qquad \dfrac{B, D \to E, \Gamma \Rightarrow D \qquad B, E, \Gamma \Rightarrow C}{B, D \to E, \Gamma \Rightarrow C} \to\text{L}}{A \to B, D \to E, \Gamma \Rightarrow C} \to\text{L}$$

This can be permuted into the following derivation:

$$\dfrac{\dfrac{A{\to}B, D{\to}E, \Gamma \Rightarrow A \quad B, D{\to}E, \Gamma \Rightarrow D}{A{\to}B, D \to E, \Gamma \Rightarrow D} \to\text{L} \qquad \dfrac{\dfrac{A{\to}B, D \to E, \Gamma \Rightarrow A}{A{\to}B, E, \Gamma \Rightarrow A} \text{Inv.} \quad B, E, \Gamma \Rightarrow C}{A{\to}B, E, \Gamma \Rightarrow C} \to\text{L}}{A \to B, D \to E, \Gamma \Rightarrow C} \to\text{L}$$

The remaining subcases proceed in an analogous manner. □

To illustrate the issue with $\vee$L (and $\mathcal{H}Split$L), and thus why the upward permutation only works for Harrop formulas, consider the following application of $\vee$L below an application of $\vee$R:

$$\dfrac{A, \Gamma \Rightarrow D \vee E \qquad \dfrac{B, \Gamma \Rightarrow D}{B, \Gamma \Rightarrow D \vee E} \vee\text{R}}{A \vee B, \Gamma \Rightarrow D \vee E} \vee\text{L}$$

The problem is that $A, \Gamma \Rightarrow D \vee E$ must be transformed into $A, \Gamma \Rightarrow D$, but $\vee$R is not invertible. Thus, one cannot permute the application of $\vee$L through the application of $\vee$R.

Theorem 4.2 (Admissibility of cut). *If* $\Gamma \Rightarrow A$ *and* $A, \Gamma' \Rightarrow C$ *are derivable, then* $\Gamma, \Gamma' \Rightarrow C$ *is also derivable.*

Proof: The proof proceeds by induction on the weight of A with a subinduction on the sum of the heights of the derivations of $\Gamma \Rightarrow A$ and $A, \Gamma' \Rightarrow C$. The proof may be organised like the proof for g3ip in [10]. If neither premise-sequent is an initial sequent, then there are three cases to consider: whether the cut-formula is not principal in the left premise-sequent, not principal only in the right premise-sequent, or principal in both premise-sequents. The cases involving the rule *Raa-at* are already described in [10, p.159], and I only need to consider the cases for the rule $\mathcal{H}Split$L. In the first two cases, one proceeds as in the case of $\vee$L. If however the cut-formula is principal in both premise-sequents and the right premise-sequent is obtained with $\mathcal{H}Split$L, then the derivation ends as follows:

$$\dfrac{\dfrac{\mathcal{H}, \Gamma \Rightarrow A \vee B}{\Gamma \Rightarrow \mathcal{H} \to A \vee B}\ \to\text{R} \qquad \dfrac{\mathcal{H} \to A, \Gamma' \Rightarrow C \qquad \mathcal{H} \to B, \Gamma' \Rightarrow C}{\mathcal{H} \to A \vee B, \Gamma' \Rightarrow C}\ \mathcal{H}Split\text{L}}{\Gamma, \Gamma' \Rightarrow C}\ \text{cut}$$

Consider now the derivation of the premise-sequent $\mathcal{H}, \Gamma \Rightarrow A \vee B$. The occurrence of $A \vee B$ in the succedent can be traced upwards in the derivation. For each branch, there is a top-most sequent after which $A \vee B$ remains parametric until the root. For this sequent, one of the following holds:

- The sequent is the conclusion-sequent of an application of $\vee$R with $A \vee B$ as the principal formula. I will refer to this as a *final* application of $\vee$R.
- The sequent is a leaf of the form $\bot, \Gamma' \Rightarrow A \vee B$. Let this be a *initial occurrence* of $A \vee B$.
- The sequent is the conclusion-sequent of an application of $\to$L where $A \vee B$ is parametric (and the branch thus continues upwards through the left premise-sequent of the application of $\to$L).

As an illustration, consider the following snippet of a derivation with three (displayed) branches:

$$\dfrac{\mathcal{H} \to \bot \vee A, \mathcal{H} \Rightarrow \mathcal{H} \qquad \dfrac{\bot \Rightarrow A \vee B \qquad \dfrac{A \Rightarrow A}{A \Rightarrow A \vee B}\ \vee\text{R}}{\bot \vee A \Rightarrow A \vee B}\ \vee\text{L}}{\mathcal{H}, \mathcal{H} \to \bot \vee A \Rightarrow A \vee B}\ \to\text{L}$$

The right-most branch has a final application of $\vee$R, the middle branch has an initial occurrence of $A \vee B$ and the occurrence of $A \vee B$ in the left-most branch is introduced through $\to$L where $A \vee B$ is parametric.

In fact, every derivation with the root $\mathcal{H}, \Gamma \Rightarrow A \vee B$ has at least one branch with either an initial occurrence of $A \vee B$ or a final application of $\vee$R. This follows by induction on the height of a derivation. If the root is a leaf, then it is itself a branch with an initial occurrence of $A \vee B$. Assume instead for the inductive step that it is obtained with another rule. If that rule is $\vee$R, then every branch of the derivation has a final application of $\vee$R. Otherwise, the sequent is obtained with $\mathcal{H}$*Split*L, $\vee$L, $\wedge$L or $\rightarrow$L. The desired result follows by applying the induction hypothesis on the premise-sequent in the case of $\wedge$L, on each premise-sequent in the case of the rules $\mathcal{H}$*Split*L and $\vee$L, and finally on the right premise-sequent in the case of $\rightarrow$L.

To simplify the rest of the proof, it is useful to transform every initial occurrence of $A \vee B$ into a final application of $\vee$R by replacing the leaf with a derivation of $\bot, \Gamma' \Rightarrow A \vee B$ from the new leaf $\bot, \Gamma' \Rightarrow A$. The result is a final application of $\vee$R because $A \vee B$ still remains parametric until the root from the inserted application of $\vee$R.

Using A/B as a metalinguistic device to represent that the formula is either A or B, it follows that each branch $\mathcal{B}$ with a final application of $\vee$R in the resulting derivation of $\mathcal{H}, \Gamma \Rightarrow A \vee B$ is of the form

$$\dfrac{\dfrac{\dfrac{\vdots}{\Gamma_{\mathcal{B}} \Rightarrow A/B}}{\Gamma_{\mathcal{B}} \Rightarrow A \vee B}\ \vee\text{R}}{\dfrac{\vdots}{\Gamma, \mathcal{H} \Rightarrow A \vee B}}$$

where $A \vee B$ remains parametric after the displayed application of $\vee$R. If $\mathcal{H}$ is not in $\Gamma_{\mathcal{B}}$, then $\mathcal{H}$ is neither an atom nor $\bot$, and there are two subcases to consider, depending on whether $\mathcal{H}$ is $D \wedge E$ or $D \rightarrow E$. Depending on the main connective, each such branch is of the following shape where the displayed application of $\vee$R is final:

$$
\begin{array}{c}
\vdots \\ \hline
\Gamma_{\mathcal{B}} \Rightarrow A/B \\ \hline
\Gamma_{\mathcal{B}} \Rightarrow A \vee B \ \ \vee\mathrm{R} \\ \hline
\vdots \\ \hline
\Gamma^*, D, E \Rightarrow A \vee B \\ \hline
\Gamma^*, D \wedge E \Rightarrow A \vee B \ \ \wedge\mathrm{L} \\ \hline
\vdots \\ \hline
\Gamma, D \wedge E \Rightarrow A \vee B
\end{array}
\qquad\qquad
\begin{array}{c}
\vdots \\ \hline
\Gamma_{\mathcal{B}} \Rightarrow A/B \\ \hline
\Gamma_{\mathcal{B}} \Rightarrow A \vee B \ \ \vee\mathrm{R} \\ \hline
\vdots \\ \hline
D \rightarrow E, \Gamma^* \Rightarrow D \qquad \Gamma^*, E \Rightarrow A \vee B \\ \hline
\Gamma^*, D \rightarrow E \Rightarrow A \vee B \ \ \rightarrow\mathrm{L} \\ \hline
\vdots \\ \hline
\Gamma, D \rightarrow E \Rightarrow A \vee B
\end{array}
$$

$\Gamma_{\mathcal{B}}$ is the context of that final application of $\vee$R at that node in the branch $\mathcal{B}$ of the derivation, and Γ^* is the context at the application of $\wedge$L or $\rightarrow$L. It is possible, but not necessary, that $\Gamma_{\mathcal{B}}$ just is Γ^*, D, E or Γ^*, E respectively. For example, if the displayed application of $\vee$R is directly above the displayed application of $\wedge$L, then $\Gamma_{\mathcal{B}}$ is Γ^*, D, E.

The above permutation lemma is now applied *iteratively* to transform the derivation into a derivation in which $\mathcal{H}$ occurs in the context of each final application of $\vee$R. For example, in the simplest case where the displayed application of $\vee$R is directly above the displayed application of $\wedge$L, one obtains the following derivation:

$$
\begin{array}{c}
\vdots \\ \hline
\Gamma^*, D, E \Rightarrow A/B \\ \hline
\Gamma^*, D \wedge E \Rightarrow A/B \ \ \wedge\mathrm{L} \\ \hline
\Gamma^*, D \wedge E \Rightarrow A \vee B \ \ \vee\mathrm{R} \\ \hline
\vdots \\ \hline
\Gamma, D \wedge E \Rightarrow A \vee B
\end{array}
$$

One can now proceed as follows for every branch $\mathcal{B}$ with a final application of $\vee$R where $\Gamma_{\mathcal{B}}*, \mathcal{H} \Rightarrow A/B$ is the premise-sequent of that final application of $\vee$R, and $\Gamma_{\mathcal{B}}* \subset \Gamma_{\mathcal{B}}$ (and $\Gamma_{\mathcal{B}}*$ just is Γ^* in the above example):

$$\dfrac{\dfrac{\dfrac{\vdots}{\Gamma_{\mathcal{B}}*, \mathcal{H} \Rightarrow A/B}}{\Gamma_{\mathcal{B}}* \Rightarrow \mathcal{H} \to A/B} \quad \mathcal{H} \to A/B, \Gamma' \Rightarrow C}{\dfrac{\Gamma_{\mathcal{B}}*, \Gamma' \Rightarrow C}{\dfrac{\vdots}{\Gamma, \Gamma' \Rightarrow C}}} \text{cut}$$

Each cut is on a formula of lesser weight. □

The next section shows how to prove interpolation for InqB using Maehara's lemma which is available because the subatom property holds for g3InqB.

5. Interpolation for InqB

A propositional logic has the interpolation property, or also, that Craig's interpolation theorem holds for it, just in case, if $A \to B$ is a theorem, then there is a formula F such that $A \to F$ and $F \to B$ are theorems and every atom in F is contained in both A and B. F is said to be an interpolant of A and B.

The standard approach to prove interpolation using sequent calculus is through Maehara's lemma. I follow here the presentation of [13]. The strategy consists in proving a more general statement by considering every partition of a sequent.

Let $\Gamma_0; \Gamma_1 \Rightarrow C$ be a partition of a sequent $\Gamma \Rightarrow C$ where Γ is Γ_0, Γ_1. The expression $\Gamma_0; \Gamma_1 \overset{F}{\Longrightarrow} C$ means that F is a split-interpolant of $\Gamma_0 \Rightarrow$ and $\Gamma_1 \Rightarrow C$, that is, if $\Gamma_0, \Gamma_1 \Rightarrow C$ is derivable then $\Gamma_0 \Rightarrow F$ and $F, \Gamma_1 \Rightarrow C$ are derivable and $\mathit{Atoms}(F) \subseteq \mathit{Atoms}(\Gamma_0) \cap \mathit{Atoms}(\Gamma_1, C)$ where *Atoms* returns the set of atoms contained in the input multiset. The aim is now to show that for every partition of a derivable sequent $\Gamma \Rightarrow C$, there is a formula F such that F is its split-interpolant. Craig's interpolation theorem is then the special case where Γ_1 is empty, Γ_0 is A and C is B.

THEOREM 5.1. *Every partition of a derivable sequent of* g3InqB *has a split-interpolant.*

Proof: The proof proceeds by induction on the height of a derivation. I present one subcase for the axioms and then the subcases involving the new rules, referring to [13] for the other details. One subcase of the axiom $\Gamma, p \Rightarrow p$ is the following partition and its split-interpolant:

$$\Gamma_0, p; \Gamma_1 \overset{p}{\Longrightarrow} p$$

After all, $\Gamma_0, p \Rightarrow p$ and $p, \Gamma_1 \Rightarrow p$ are derivable and the atom p is in the intersection of $Atoms(\Gamma_0, p)$ and $Atoms(\Gamma_1, p)$. For the inductive step with the rule *Raa-at*, there is only one subcase to consider. Moreover, it is trivial to see that whatever is a split-interpolant for the premise is also a split-interpolant for the conclusion:

$$\frac{\Gamma_0; \neg p, \Gamma_1 \overset{F}{\Longrightarrow} \bot}{\Gamma_0; \Gamma_1 \overset{F}{\Longrightarrow} p} \text{ Raa-at}$$

Assume that $\Gamma_0 \Rightarrow F$ and $F, \neg p, \Gamma_1 \Rightarrow \bot$ is derivable, then $F, \Gamma_1 \Rightarrow p$ is also derivable. There is no change in atoms from premise to conclusion.

In the case of $\mathcal{H}Split$L, there are two subcases depending on the location of the principal formula. The first subcase goes as follows:

$$\frac{\Gamma_0; \mathcal{H} \to A, \Gamma_1 \overset{F}{\Longrightarrow} C \qquad \Gamma_0; \mathcal{H} \to B, \Gamma_1 \overset{G}{\Longrightarrow} C}{\Gamma_0; \mathcal{H} \to A \vee B, \Gamma_1 \overset{F \wedge G}{\Longrightarrow} C} \ \mathcal{H}Split\text{L}$$

This holds because one can proceed as follows after applying the inductive hypothesis on the premise-sequents:

$$\frac{\Gamma_0 \Rightarrow F \qquad \Gamma_0 \Rightarrow G}{\Gamma_0 \Rightarrow F \wedge G} \qquad \frac{\dfrac{F, \mathcal{H} \to A, \Gamma_1 \Rightarrow C \qquad G, \mathcal{H} \to B, \Gamma_1 \Rightarrow C}{F, G, \Gamma_1, \mathcal{H} \to A \vee B, \Gamma_1 \Rightarrow C}}{F \wedge G, \Gamma_1, \mathcal{H} \to A \vee B, \Gamma_1 \Rightarrow C}$$

The atoms of $F \wedge G$ are in the intersection of Γ_0 and $\Gamma_1, \mathcal{H} \to A \vee B, C$ since those of F are in the intersection of Γ_0 and $\Gamma_1, \mathcal{H} \to A, C$ while those of G in the intersection of Γ_0 and $\Gamma_1, \mathcal{H} \to B, C$. For the other subcase, the split-interpolant is $F \vee G$. □

This establishes that every partition of every derivable sequent of g3InqB has a split-interpolant. It follows that InqB has the interpolation property.

Basic Inquisitive Logic is an intermediate logic, that is, a logic inclusively between intuitionistic and classical logic, and it was established in [8] that there are only seven intermediate logics that satisfy Craig's interpolation theorem, including intuitionistic and classical logic. The theorem presented in this section does not contradict that result. Instead, the result in [8] concerns logics that satisfy uniform substitution, which Basic Inquisitive Logic does not.

6. Conclusions

I have in this paper presented a sequent calculus for InqB obtained by expanding g3ip for IPL with suitable rules for the Split property and double negation elimination for atoms, established that cut is admissible for the sequent calculus, and demonstrated that it can be used to show that InqB has the interpolation property.

The proof-theoretic approach presented in this paper can also be employed to provide a syntactic decidability proof for InqB. This will proceed more or less in the same way as described in [10, p. 45] for the corresponding sequent calculus g3ip for IPL. It will use the same halting condition regarding the reduction of conditionals in the antecedent which are not of the form $\mathcal{H} \to (A \vee B)$. Formulas of that form will instead be reduced with $\mathcal{H}$*Split*L. They should, like other invertible rules, be reduced prior to succedent disjunctions to avoid unnecessary back-tracking. The additional complexity for the decidability of InqB beyond that of IPL consists thus in determining whether the antecedent of a conditional with a disjunctive consequent is a Harrop formula or not. But this is linear on the length of the antecedent.

I leave two interesting questions about the sequent calculus g3InqB presented in this paper for future research:

- Can a countermodel in the support semantics for InqB be extracted from a failed proof search?

- Can it be expanded to a sequent calculus for first-order Inquisitive Logic, InqBQ?

There is currently no deductive system which is known to be sound and complete with regard to InqBQ, and it is also not known whether entailment in InqBQ is compact [3, p. 157]. A positive answer to the first question would encourage investigations into the properties of the sequent calculus obtained by expanding g3InqB with appropriate sequent calculus rules based on the sound (but possibly incomplete) natural deduction system for InqBQ presented by [3].

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References

[1] A. Anttila, **Not Nothing: Nonemptiness in Team Semantics**, Ph.D. thesis, Universiteit van Amsterdam (2025).

[2] J. Chen, M. Ma, *Labelled Sequent Calculus for Inquisitive Logic*, [in:] **Logic, Rationality, and Interaction** (2017), pp. 526–540, DOI: https://doi.org/10.1007/978-3-662-55665-8_36.

[3] I. Ciardelli, **Inquisitive Logic: Consequences and Inference in the Realm of Questions**, Springer, Cham (2022), DOI: https://doi.org/10.1007/978-3-031-09706-5.

[4] I. Ciardelli, J. Groenendijk, F. Roelofsen, **Inquisitive Semantics**, Oxford University Press, Oxford, England (2018), DOI: https://doi.org/10.1093/oso/9780198814788.001.0001.

[5] I. Ciardelli, R. Iemhoff, F. Yang, *Questions and Dependency in Intuitionistic Logic*, **Notre Dame Journal of Formal Logic**, vol. 61(1) (2020), pp. 75–115, DOI: https://doi.org/10.1215/00294527-2019-0033.

[6] I. Ciardelli, F. Roelofsen, *Inquisitive Logic*, **Journal of Philosophical Logic**, vol. 40(1) (2011), pp. 55–94, DOI: https://doi.org/10.1007/s10992-010-9142-6.

[7] R. Harrop, *Concerning Formulas of the Types $A \to B \lor C$, $A \to (Ex)B(x)$*, **Journal of Symbolic Logic**, vol. 25(1) (1960), pp. 27–32, DOI: https://doi.org/10.2307/2964334.

[8] L. L. Maksimova, *Craig's theorem in superintuitionistic logics and amalgamable varieties of pseudo-boolean algebras*, **Algebra and Logic**, vol. 16 (1977), p. 427–455, DOI: https://doi.org/10.1007/BF01670006.

[9] V. Müller, **On the Proof Theory of Inquisitive Logic**, Master's thesis, Universiteit van Amsterdam (2023).

[10] S. Negri, J. von Plato, **Structural Proof Theory**, Cambridge University Press, New York (2001).

[11] I. Pezlar, *Constructive Validity of a Generalized Kreisel–Putnam Rule*, **Studia Logica**, vol. 113 (2025), p. 1367–1399, DOI: https://doi.org/10.1007/s11225-024-10129-x.

[12] V. Punčochár, *A Generalization of Inquisitive Semantics*, **Journal of Philosophical Logic**, vol. 45(4) (2016), pp. 399–428, DOI: https://doi.org/10.1007/s10992-015-9379-1.

[13] A. S. Troelstra, H. Schwichtenberg, **Basic Proof Theory**, Cambridge University Press, New York (2000), DOI: https://doi.org/10.1017/CBO9781139168717.

[14] J. von Plato, *A Proof of Gentzen's Hauptsatz Without Multicut*, **Archive for Mathematical Logic**, vol. 40(1) (2001), pp. 9–18, DOI: https://doi.org/10.1007/s001530050170.

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